Orbital Formationkeeping with Differential Drag

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This paper examines the feasibility of using differential drag between two spacecraft as the means for controlling their relative positions. The equations of relative motion between two satellites are derived, and a coordinate transformation is made to reduce the formationkeeping problem to the simultaneous solution of a double integrator and a harmonic oscillator. A simple feedback control law is developed that simultaneously and dependently solves the double integrator and harmonic oscillator; the control law consists of two parts: the main control law and the eccentricity-minimizing control scheme. Results are presented of four test cases, which show that the control law can drive a satellite from an initial position to a target position and maintain the satellite at that location.

Nomenclature

a = magnitude of differential drag acceleration
e = eccentricity of elliptic motion
n = mean orbital angular velocity of master satellite
r = radius of locus in $(\alpha,\beta/2)$ plane during eccentricity minimizing control
t = time
x,y = radial and circumferential positions, respectively, of slave satellite relative to target position
$\bar{x},\bar{y}$ = average radial and circumferential position offsets, respectively, of slave satellite relative to target position
α,β = radial and circumferential components, respectively, of harmonic motion due to eccentricity
δ,Φ = angles defined in Fig. 6 for the eccentricity minimizing control switch algorithm
$\Theta_0,\Theta_1,\Theta_2$ = angles traversed in the $(\alpha,\beta/2)$ plane during the three phases of the eccentricity minimizing control

Introduction

SEVERAL requirements exist for spacecraft to fly in specified positions relative to other space vehicles. Space tugs and orbital transfer vehicles may have to place themselves at, and maintain certain positions with respect to, the space station until they are required to perform some task. Space shuttles with replacement crews and new supplies may also have to orbit in specified positions relative to the station before they are allowed to dock. When one satellite must fly in a certain position with respect to another, the two are flying in formation; formationkeeping refers to the active measures taken to keep them in their relative positions. Two aspects of the formation flying problem are considered: the maneuvering of one spacecraft relative to another and the actual formationkeeping.

Controllers for formationkeeping were developed by Vassar and Sherwood[1] and Redding et al.[2] Both studies assumed that the actuators for formationkeeping were chemical thrusters; the jets provided sufficiently high thrust to allow the control to be treated as impulsive changes in velocity. Thrust was available along all three translational axes. The control laws were developed using optimal control theory; the first study optimized thruster firing frequency and the second minimized fuel consumption.

By convention, the satellite that takes active control measures to remain in a specific location relative to another spacecraft is called the slave satellite; the spacecraft on which the slave is flying formation is called the master satellite. When differential drag is used to formationkeep, both vehicles may take active measures to retain their relative positions; in this case, one satellite is arbitrarily deemed the master and the other the slave.

Differential drag between a pair of satellites is the difference in drag per unit mass acting on each of the satellites. If the satellites are passing through similar density atmospheres with similar velocities, then any differential drag is due to different ballistic coefficients of the vehicles.

In this paper, it is assumed that atmospheric density is uniform and that the velocities and ballistic coefficients of the satellites are initially equal. Differential drag is created through the use of drag plates attached to both of the satellites. The angle of attack of each drag plate can be chosen to be 0 or 90 deg, resulting in on-off control. When the drag plate of both spacecraft are at a 0- or 90-deg angle of attack, no differential drag is created; this is referred to as zero differential drag. Setting the angle of attack of the drag plate connected to the master satellite at 90 deg and the angle of attack of the plate connected to the slave at 0 deg creates positive differential drag and acts as a force along the velocity vector. Setting the angle of attack of the plate connected to the master at 0 deg and the angle of attack of the plate connected to the slave at 90 deg creates negative differential drag and acts as a force opposite to the velocity vector. The magnitude of the differential drag acceleration created was estimated to be 54×10^{-6} ft/s^2; this value was calculated by using drag information obtained from skylab data,[3] a 13×270 ft drag plate as sized in Ref. 4, and a formationkeeping altitude of 270 n.m.

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Reasons to use differential drag as an actuator in formationkeeping include:

1) Plumes from jet firings may impinge on the master satellite, imparting some undesired motion or contaminating solar panels.

2) An impulsive jet firing may disturb a manufacturing process on the slave satellite; the accelerations created by differential drag are much smaller than those that arise from jet firings.

3) Fuel savings are possible.

Additional assumptions made in the formulation of the problem are that a 0- or 90-deg angle of attack of the drag plate does not produce torques on the satellite, that the angle of the drag plate can be changed instantaneously, and that no lift is generated.

Equations of Motion

The desired position of the slave satellite with respect to the master will be the origin of the target reference coordinate system, shown in Fig. 1. It is assumed that the origin is a reference circular orbit. The x direction will be measured along the radius vector to the target from the center of the Earth, with the direction away from the Earth positive. The y direction will be measured along the velocity vector of the target. The third axis, z, is normal to the orbit plane, forming a right-handed coordinate system.

Since the relative separation of the two satellites is small compared to the orbital radius, the linearized equations of Hill[5] are valid for a circular orbit.

$$\ddot{x} = 2n\dot{y} + 3n^2x + F_x/m \tag{1}$$

$$\ddot{y} = -2n\dot{x} + F_y/m \tag{2}$$

where n is the mean orbital angular motion of the master satellite, F_x and F_y are the forces applied to the satellite in the x and y directions, respectively, and m is the satellite mass. F_x is equal to zero as no lift is generated, and F_y is equal to the differential drag; it is assumed that the values the differential drag can have are a, 0, or $-a$. When the master satellite has the greater drag, a is positive; with equal drag, a is zero; when the slave satellite has the greater drag, a is negative.

Unforced solutions to these equations have been presented by several authors[6–8] and are given in state variable form as follows:

$$\begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 4-3\cos nt & 0 & \frac{1}{n}\sin nt & -\frac{2}{n}\cos nt + \frac{2}{n} \\ 6\sin nt - 6nt & 1 & \frac{2}{n}\cos nt - \frac{2}{n} & \frac{4}{n}\sin nt - 3t \\ 3n\sin nt & 0 & \cos nt & 2\sin nt \\ 6n\cos nt - 6n & 0 & -2\sin nt & 4\cos nt - 3 \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \\ \dot{x}_0 \\ \dot{y}_0 \end{bmatrix}$$

The unforced orbit is a 2×1 ellipse whose center maintains a constant altitude difference $\bar{x}$ with the target position; the center of the ellipse moves ahead of or behind the target position with a relative velocity $\dot{\bar{y}}$ that is a function of the average altitude difference. If the average position is given as

$$\bar{x} = 4x + 2\dot{y}/n \tag{3}$$

$$\bar{y} = y - 2\dot{x}/n \tag{4}$$

We can express motion relative to this average position as

$$\alpha = -3x - 2\dot{y}/n = x - \bar{x} \tag{5}$$

$$\beta = 2\dot{x}/n = y - \bar{y} \tag{6}$$

The eccentricity of the orbit is given by

$$e \equiv [\alpha^2 + (\beta/2)^2]^{\frac{1}{2}} \tag{7}$$

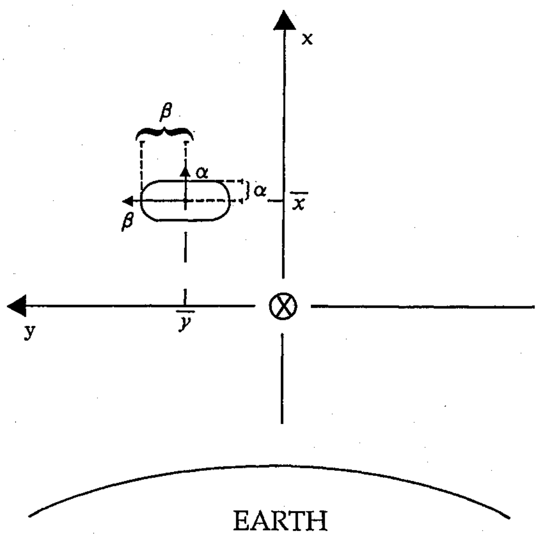


Fig. 1 Target reference coordinate system.

The transformation leads to two uncoupled, second-order, linear differential equations

$$\ddot{\bar{y}} = -3a \tag{8}$$

$$\ddot{\beta} + n^2\beta = 4a \tag{9}$$

Controlling $\bar{y}$ and β effectively controls $\bar{x}$ and α as

$$\bar{x} = -2\dot{\bar{y}}/(3n) \tag{10}$$

and

$$\alpha = -\dot{\beta}/(2n) \tag{11}$$

Equation (8) is a double integrator, and Eq. (9) is a harmonic oscillator. The forced solutions are obtained as

$$\bar{x}(t) = \bar{x}_0 + 2at/n \tag{12}$$

$$\bar{y}(t) = \bar{y}_0 - 3n\bar{x}_0t/2 - 3at^2/2 \tag{13}$$

$$\alpha(t) = \alpha_0 \cos nt + \left(\frac{\beta_0}{2} - \frac{2a}{n^2}\right)\sin nt \tag{14}$$

$$\frac{\beta}{2}(t) = \left(\frac{\beta_0}{2} - \frac{2a}{n^2}\right)\cos nt - \alpha_0 \sin nt + \frac{2a}{n^2} \tag{15}$$

Time optimal control for each system alone is well known,[9,10] using switch curves in the phase plane. However, here the two control laws must be combined in such a way that all four states are driven to the origin simultaneously.

Effect of Control in the Phase Plane

In the $(\bar{x},\bar{y})$ plane, the forced solutions plot as parabolas. Positive differential drag causes the state to move along the trajectories represented by the solid lines in Fig. 2; negative differential drag causes the state to move along the dashed trajectories. Zero differential drag would cause the state to move parallel to the $\bar{y}$ axis at a rate that is a function of $\bar{x}$; the movement would be in the positive $\bar{y}$ direction if $\bar{x}$ were positive and in the negative $\bar{y}$ direction if $\bar{x}$ were negative.

In the (α,β) plane, the loci are 2×1 ellipses, centered on the points $(4a/n^2,0)$ and $(-4a/n^2,0)$ caused by positive and negative drag, respectively. By plotting $\beta/2$ instead of β, the loci become circles centered on the points $(2a/n^2,0)$ and $(-2a/n^2,0)$ with a radius arm of magnitude e that rotates at the angular rate n. Positive differential drag causes the state to move along the trajectories represented by the solid lines in Fig. 3; negative differential drag causes the state to move along the dashed

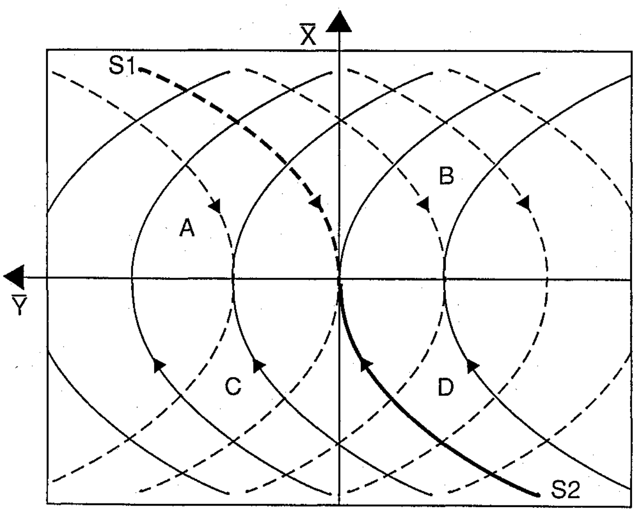


Fig. 2 Control trajectories in the $\bar{x}$, $\bar{y}$ plane.

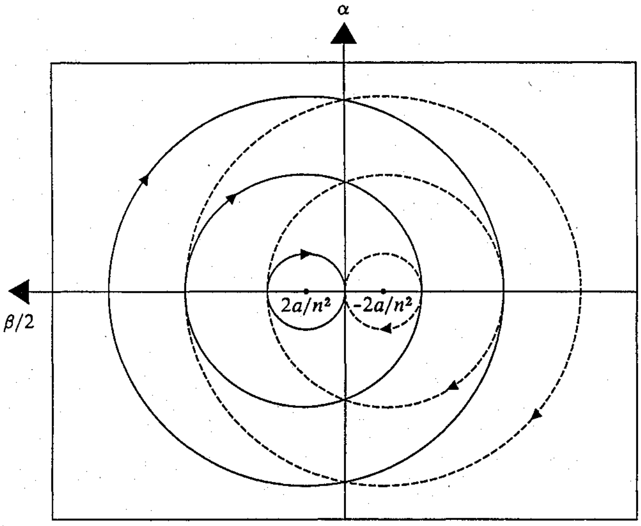


Fig. 3 Control trajectories in the $\alpha,\beta/2$ plane.

trajectories. Zero differential drag would cause the state to circle about the origin.

When control is switched, the instantaneous values of the four state variables remain constant, but future values are determined by the appropriate new trajectory.

The primary interest in this application centered on feasibility rather than optimality. The objective was to demonstrate that a relatively simple control system without extensive iterations would actually work. This was carried out in Ref. 11 and will be presented in abbreviated form in the remainder of this paper.

Formation Maneuvering

A two-part control law was developed to drive the time average relative position to zero and reduce the eccentricity as much as possible. If the eccentricity is driven to zero while the average relative position is nonzero, each spacecraft will be in a circular orbit, but the radii may not be matched, causing the spacecraft to separate. The control strategy used is to first bring the average position of the slave to that of the target while decreasing eccentricity as much as feasible; this process is called the main control law. After the average position has been brought to zero, the eccentricity is further reduced using the eccentricity-minimizing control scheme.

Main Control Law

Once the average position is moving toward the origin, the control law used differs from the double integrator minimum-time solution[9] in order to reduce eccentricity.

If the average position lies in areas B or C as shown in Fig. 2, then the sign of the control is chosen to move the state along a parabola toward the appropriate switch curve. This constant control in the B or C region causes the state to circle about the appropriate point in the $(\alpha,\beta/2)$ plane. For example, if the average position were in area B, a negative differential drag would be commanded to move the average position toward the D region in the $(\bar{x},\bar{y})$ plane; the resultant trajectory in the $(\alpha,\beta/2)$ plane would be a circle around $(-2a/n^2,0)$.

The control will be switched when $\bar{y} > 0$ and $\bar{x} <$ S1 or when $\bar{y} < 0$ and $\bar{x} >$ S2. Because

$$\frac{d(e^2)}{dt} = \frac{-4a\alpha}{n} \tag{16}$$

the eccentricity is decreased whenever the control a has the same sign as α. For a to have the same sign as α, a must change sign when α changes sign, which is twice each orbit. As a result, the state moves in a sawtooth pattern toward the appropriate switch curve in the $(\bar{x},\bar{y})$ plane. For the sawtooth action to move the slave toward the target, the average position during the sawtooth must stay in area A or D.

An exception to a changing sign twice each orbit occurs when the eccentricity is small. When $e < 4a/n^2$, the sawtooth maneuver can at best bound the eccentricity; this is done by a switching control once per orbit and is explained in detail in Ref. 11.

Once the switch curve in the $(\bar{x},\bar{y})$ plane is reached, the control that brings the state to the origin is exercised immediately, no matter what the consequence to the eccentricity. After the origin is reached, the eccentricity-minimizing control scheme is used to reduce any remaining eccentricity.

Eccentricity-Minimizing Control Scheme

The eccentricity-minimizing control scheme was designed to reduce the eccentricity as much as possible without altering the final average position of the slave. It approximates the harmonic oscillator minimum-time solution.[10] In this scheme, the commanded differential drag is initially zero, which keeps the average position of the slave at the target and results in the state circling about the origin in the $(\alpha,\beta/2)$ plane.

A hat-shaped maneuver is then performed in the $(\bar{x},\bar{y})$ plane. The first two legs of the hat are the trajectories caused by opposite commanded controls; a slight difference in the duration of the controls causes an offset from the $\bar{y}$ axis at the end of the second leg, as can be seen in Fig. 4. The base of the hat is formed by the unforced motion or drift of the state caused by the position offset $\bar{x}$; during this phase of the eccentricity-minimizing control scheme, the differential drag is zero.

The angle traversed by the state in the $(\alpha,\beta/2)$ plane while the differential drag is initially zero is Θ_0. The angle traversed by the state during the first interval of nonzero commanded drag will be referred to as Θ_1; the angle corresponding to the subse-

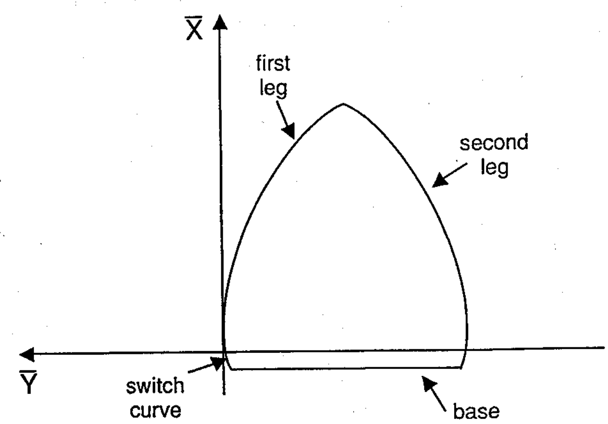


Fig. 4 Eccentricity-reducing maneuver.

quent opposite drag will be known as Θ_2, as shown in Fig. 5. For the eccentricity-minimizing control scheme to work, the second leg of the hat in the $(\bar{x},\bar{y})$ plane must be longer than the first leg, so that the state crosses the $\bar{y}$ axis and the coasting orbital motion brings the average position back toward the origin. For the second leg to be longer than the first leg, the angle that the second leg traverses in the $(\alpha,\beta/2)$ plane must be larger than the angle of the first leg; Θ_2 must be larger than Θ_1.

The difference between Θ_1 and Θ_2 affects the closing rate of the slave during the drift phase of the hat maneuver and the minimum eccentricity that the control law can achieve. For $e > 4a/n^2$, $\Theta_2 - \Theta_1$ was set to be 20 deg to insure a large closing rate of the average position of the slave toward the target in the unthrusted phase of the maneuver. For $e < 4a/n^2$, the difference was chosen as 10 deg to allow an acceptable final eccentricity.

For $e > 4a/n^2$, Θ_2 is given as 180 deg by the phase plane for the harmonic oscillator; thus, Θ_1 equals 160 deg and Θ_0 is 20 deg. The commanded drag will switch from zero to either plus or minus a when Θ_0 equals 20 deg. For α greater than zero, the proper control will be positive. This first switch will reduce the eccentricity and form the first leg of the hat. The next switch will occur at the $\beta/2$ axis after the angle Θ_1 has been passed; it will command a change in sign. This action will reduce the eccentricity once again and form the second leg of the hat. When the $\beta/2$ axis is reached again (corresponding to a transferred Θ_2 of 180 deg), the drag is commanded to be zero. The states in the $(\alpha,\beta/2)$ plane will then circle about the origin, and the states in the $(\bar{x},\bar{y})$ plane will drift toward the origin, forming the base of the hat.

When the drifting slave encounters the switch curve in the $(\bar{x},\bar{y})$ plane, the appropriate drag control will drive the average position of the slave to the target. The strategy will be repeated until the eccentricity has been reduced enough to go through the eccentricity-minimizing control scheme for $e < 4a/n^2$.

For $e < 4a/n^2$, the eccentricity of the system determines the values of Θ_1, Θ_2, and the Θ_0 at which the first switch to nonzero drag will occur. The algorithm to select the proper values of Θ_1, Θ_2, and Θ_0 follows from the geometry in Fig. 6.

$$\Theta_0 = \tan^{-1}(2\alpha/\beta) \tag{17}$$

$$r = [e^2 + (2a/n^2)^2 - 2e(2a/n^2)\cos\Theta_0]^{\frac{1}{2}} \tag{18}$$

$$\delta = \sin^{-1}[(e/r)\sin\Theta_0] \tag{19}$$

$$\Phi = \cos^{-1}\left(\frac{r^2 + (4a/n^2)^2 - (2a/n^2)^2}{2r(4a/n^2)}\right) \tag{20}$$

$$\Theta_1 = \delta + \Phi \tag{21}$$

$$\Theta_2 = \sin^{-1}\left(\frac{r\sin\Phi}{2a/n^2}\right) \tag{22}$$

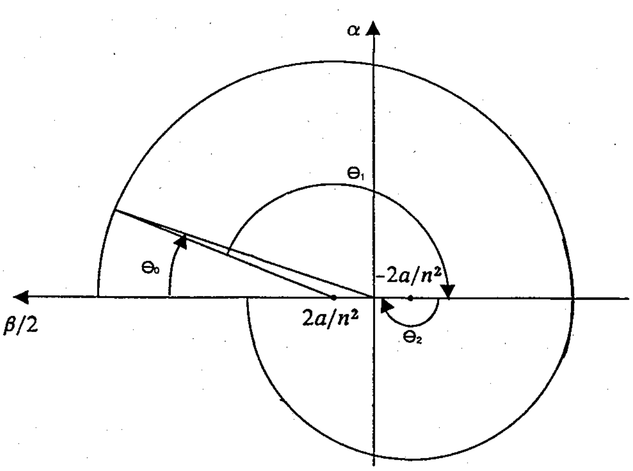


Fig. 5 $(\alpha,\beta/2)$ trajectory during eccentricity reduction.

The first switch to nonzero differential drag is made when Θ_2 is 10 deg greater than Θ_1. The second switch occurs when the current value of e satisfies

$$e^2 = 2(2a/n^2)^2 - 2(2a/n^2)^2\cos\Theta_2 \tag{23}$$

When the eccentricity reaches zero, the drift phase begins. A switch is made to zero commanded drag; the eccentricity stays at zero, and the unthrusted motion of the slave corresponds to the base of the hat in the $(\bar{x},\bar{y})$ plane.

When the state crosses the switch curve in the $(\bar{x},\bar{y})$ plane, the appropriate drag control drives the average position of the slave to the target. After the average position reaches that of the target, the final eccentricity should be acceptable as determined by the choice of the 10-deg difference between Θ_2 and Θ_1.

Maneuver Simulations

Simulations were performed on the Space Systems Simulator at the Charles Stark Draper Laboratory, an engineering tool used in support of Shuttle, space station, and Department of Defense payloads. The motion of both the target and the slave were integrated in spherical—then converted into inertial—coordinates; the position and velocity of the target and slave were differenced to calculate $\bar{y}$, $\bar{x}$, α, and β. The magnitude of the differential drag created by a 90-deg deflection of the drag plate was computed once using a Jacchia atmosphere and was assumed constant thereafter.

Eight sets of initial conditions were tested. The control scheme worked satisfactorily in all cases. A typical case is shown. Figures 7–9 show the (x,y) plane, $(\alpha,\beta/2)$ plane, and the

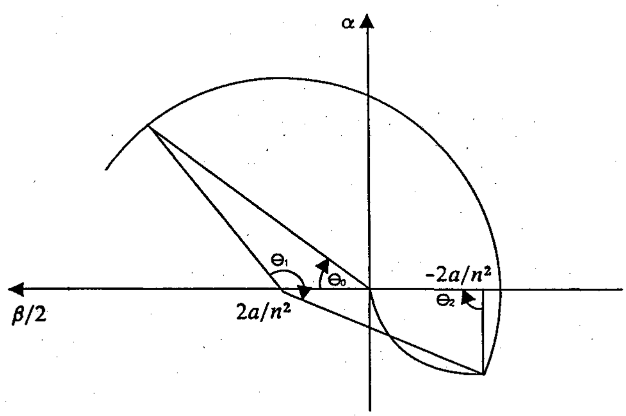


Fig. 6 Selection of Θ_0, Θ_1, Θ_2 for eccentricity minimization.

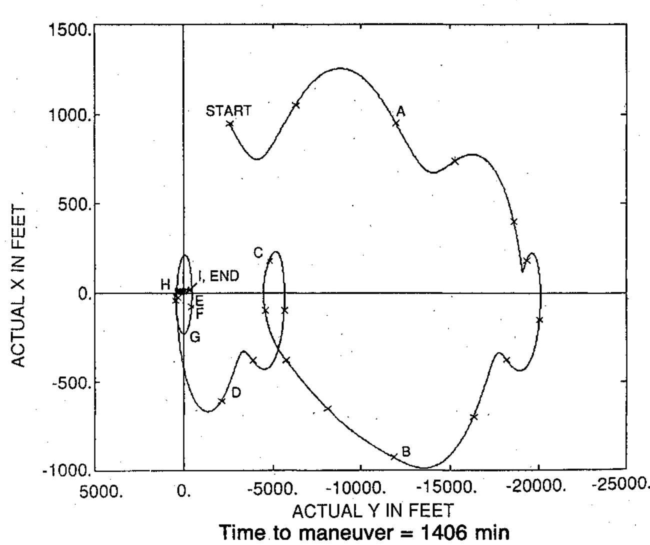
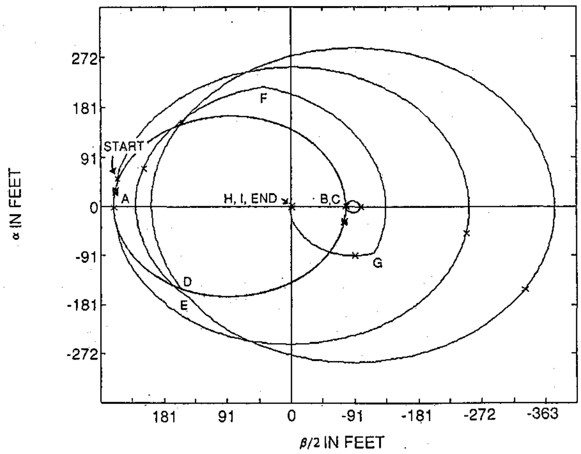
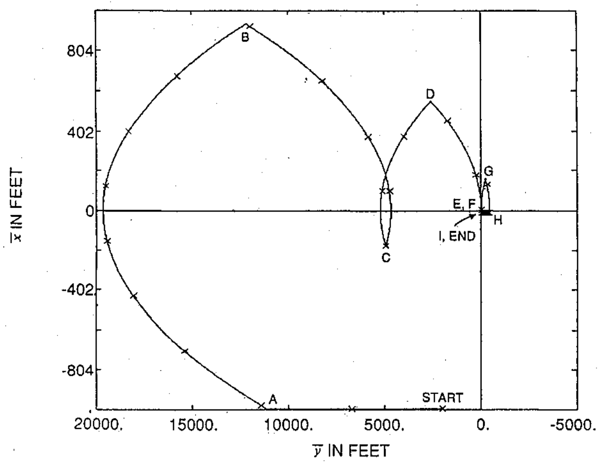


Fig. 7 (x,y) plane in example maneuver.



Time to maneuver = 1406 min

Fig. 8 $(\alpha, \beta/2)$ plane in example maneuver.



Time to maneuver = 1406 min

Fig. 9 $(\bar{x}, \bar{y})$ plane in example maneuver.

motion in the $(\bar{x}, \bar{y})$ plane, respectively, for a given case to illustrate the separate control laws and their final effect on the actual motion of the spacecraft. Other test cases produced similar results; details of all of the results are available in Ref. 11.

Position Maintenance Simulations

Two methods were used to keep the slave at the target position using differential drag. One simply restarted the control scheme once the slave had drifted 10 ft away from the origin. The other used two rules, collectively referred to as the limit cycle control law: 1) if $\bar{x}$ is negative when $\bar{y}$ becomes more than 1 ft away from the origin, a positive drag is commanded for one 0.4-min time step; 2) if $\bar{x}$ is positive when $\bar{y}$ becomes more than 1 ft away from the origin, a negative drag is commanded for one 0.4-min time step. The acceptable positions for $\bar{x}$ after it is placed at the origin by the formation maneuvering control laws are such that the proper drag, commanded to last for 0.4 min, will send $\bar{x}$ to the other side of the $\bar{y}$ axis; this maneuver will reverse the direction in which the slave is drifting with respect to the target.

Three sets of initial conditions were tested for both control methods. Figure 10 shows the results when the formation maneuvering control law is restarted; Fig. 11 presents the results for the limit cycle control law. The other test cases are similar; details of all the results are available in Ref. 11.

- Restart control system when position exceeds a given threshold

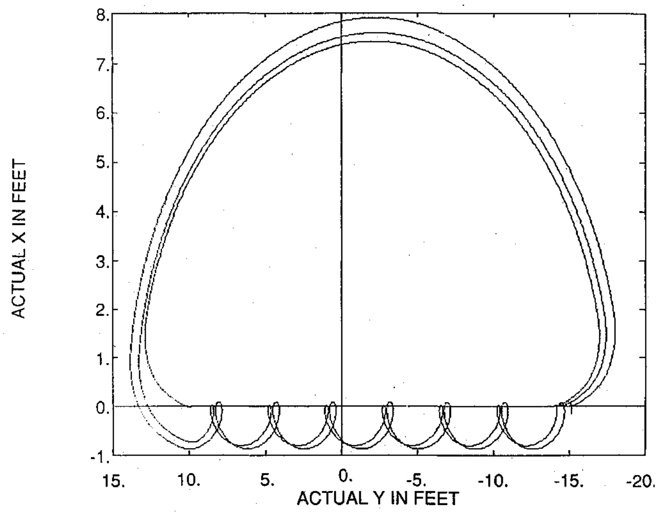


Fig. 10 Maintaining relative position.

- For $\bar{x} < 0$, $|\bar{y}| > db$, apply a for a fixed interval
- For $\bar{x} > 0$, $|\bar{y}| > db$, apply $-a$ for a fixed interval

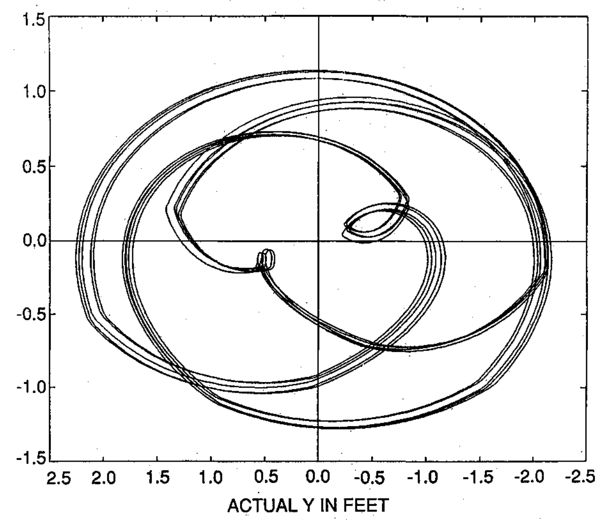


Fig. 11 Limit cycle controller.

Conclusions

The formationkeeping problem has been formulated as the simultaneous solution of a double integrator and a harmonic oscillator. The double integrator models the average position of the slave relative to the target; the harmonic oscillator models the eccentricity of the slave. Solving the double integrator and harmonic oscillator concurrently leads to the positioning of the slave at the target.

The main control law drives the average position of the slave to that of the target while minimizing eccentricity as much as possible. The eccentricity-minimizing control scheme is activated once the average position of the slave is at the target; its purpose is to reduce the eccentricity of the slave as much as possible without jeopardizing its final average position.

The control law was tested using eight sets of initial conditions and two different simulations. The control law was capable of moving a slave satellite from an arbitrary initial position to a specified orbiting target. All cases tested were driven to the target in less than 24 h even though the control did not represent the minimum time solution.

Two methods were tested for position maintenance. One activated the main control law when the slave drifted a certain distance from the origin; the other used the limit cycle control law. The main control law was able to keep the slave within 18 ft of the origin while the limit cycle control law kept the slave within 4.17 ft of the origin.

The time optimal solution could be found for the formation flying problem for a limited range of e. Assuming k segments of time that have alternating signs of the control, successive applications of Eqs. (12–15) give the final state as a function of the initial state and the k time segments; using the origin as the final state and knowing the initial position, this iterative process can be used to find the optimum duration of the k time segments.

A linear controller (with saturation) could be designed for position maintenance if the angle of attack of the drag plates was assumed to vary continuously from −90 to 90 deg.

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